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Particle-Particle Collective Excitation of Sn isotopes

Ali H. Taqi and Fahema A. Saber

Department of Physics, College of Science, Kirkuk University,
Kirkuk, Iraq

***Abstract*—In this paper, energy-level schemes and reduced electric transition strengths of neutron-rich Tin isotopes ^{102,110,116,120,122}Sn (Z=50) are studied using collective models, that is, particle-particle Tamm-Dancoff Approximation and particle-particle Random Phase Approximation. According to these models, the excited states of closed-core *A*+2 systems with multipolarity *J* and isospin *T* can be described as a linear combination of particle-particle pairs. In our investigation, the low-lying states of the investigated isotopes ^{102,110,116,120,122}Sn are described by acting two-particle operators on a correlated core ^{100}Sn, ^{108}Sn, ^{114}Sn, ^{118}Sn, and ^{120}Sn, respectively. The Hamiltonian is diagonalized within the model space include {$1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$} orbits, using the matrix elements of neutron-neutron interaction and modified surface delta interaction. The calculated values are checked by using the resultant eigenvalues and eigenvectors to calculate the excitation energies and reduced electric transition strengths. Our calculated results are compared to the available experimental data, and these comparisons led to reasonable agreements. Effective charges are also used to account for the core polarization effect.**

***Index Terms*—Collective excitations, Energy-level schemes, Particle-particle Random Phase Approximation; Particle-particle Tamm-Dancoff Approximation.**

I. Introduction

Elsasser in 1934 observed that there are some nucleon numbers correspond to greater stability (given the name magic numbers) than other numbers (Sorkin, 2014). Subsequently, Mayer (1949); Haxel, Jensen and Suess (1949), indicated that the nuclear potential could be constituted by a one-body Harmonic Oscillator with a spin-orbit potential for creating the shell gaps at 8, 20, 28, 50, 82, and 126 (Elliott and Lane, 1954).

The main goal of theoretical nuclear physics is to develop a universal approach that can describe the excited state of all nuclear systems with the same accuracy. Nuclear structure problems, as in many branches of physics (theoretical atomic, solid-state, nuclear, and elementary particle physics), are many-body problems, which can cause the dimension of Hilbert space to grow rapidly as the number of particles increases, and the dimension of such space becomes very large in many cases, preventing complete calculations. With such systems, the calculation of all possible truncated model spaces and various approximate approaches exists (Rowe, 1970; Brussaard and Glaudemans, 1977; Ring and Schuck, 1980).

The nuclear structure can be described according to the nuclear shell model where each nucleon in moving in single-particle orbits within some potential and regulates the energy levels in terms of quantum numbers (n□□) (Bhatt, Nestor Jr. and Raman, 1992). The nuclear shell model treats nucleons as occupying various orbits of a single particle orbits that are occupied and bound according to the principle of Pauli exclusion (Nichols, 2014; Tajima and Suzuki, 2001; Hasan, Obeed and Rahim, 2020).

There are many correlations that cannot be reproduced using a simple shell model and Hartree-Fock (HF) calculations. These correlations can be taken into account by breaking the HF core and raising a nucleon from below to above Fermi level (Taqi, 2013; Rowe, 1970). According to the collective models, the excited states of A+2 nuclei can be described as a linear combination of particle-particle (*pp*) pairs (Ring and Schuck, 1980; Heyde, 1994). Such approximations are called particle-particle Tamm-Dancoff approximation (*pp* TDA). A more general model can be obtained by treating the ground and the excited states more symmetrically, that is, both the ground states and the excited states can be described as a linear combination of particle-particle and hole-hole states. Such an approximation is referred to as the particle-particle random phase approximation (*pp* RPA) (Taqi, 2007; Taqi, Rasheed and Amin, 2010).

Several theories and interaction have been used to explain the shell model calculations of Sn isotopes, for neutron-deficient Sn isotopes (Engeland, et al., 1995; Covello, et al., 1997; Schubert, et al., 1995) taking ^{100}Sn as core, for ^{106,107,108,109}Sn isotopes using CD-Bonn and Nijmegen1 interactions (Dikmen, 2009), for exotic $^{134.136.138,140}$Sn isotopes with a realistic effective interaction (Covello, et al., 2011), for even $^{102\text{-}108}$Sn and odd $^{103\text{-}107}$Sn isotopes using different interactions (Trivedi, et al., 2012), for ^{104,106,108}Sn based on the CD-Bonn nucleon-nucleon (N-N) interaction (Jassim, 2013), and for even-even $^{100\text{-}108}$Sn isotopes by with the effective interactions Snet, SN100PN Delta interaction (Al-Attiah, Majeed and Al-Kawwaz, 2013).

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Corresponding author's e-mail: alitaqi@uokirkuk.edu.iq

The energy levels, binding energy, and reduced transition probabilities B(E2;$0^+ \rightarrow 2^+$) were calculated for even-even ^{134,136}Sn, and ^{134,136}Te around doubly magic core ^{132}Sn using shell model code Nushellx@MSU for Windows and employing the effective interactions jj56pna, jj56pnb, kh5082, cw5082, jj56cdb, and khhe (Majeed and Obaid, 2016). The surface delta interaction (SDI) and modified SDI (MSDI) are used by applying the nuclear shell model to calculate values of excitation energies for isotopes of equal mass number containing two nucleons outside the closed core ^{114}Sn, these nuclei are that the isotope ^{116}Sn contains two neutrons within the model space ($3s_{1/2}$, $2d_{3/2}$, $1h_{11/2}$) and the other isotope is that ^{116}Te contains two protons within the model space $1g_{7/2}$, $2d_{5/2}$, $3s_{1/2}$, $2d_{3/2}$, $1h_{11/2}$ (Obeed and Abed, 2020).

Theoretical nuclear physics is the development of models for describing structure and properties of atomic nuclei. A successful model must reasonably well account for previously measured nuclear properties and must predict additional properties that can be measured in new experiments. Therefore, in this paper, particle-particle excitations of some even-even Tinisotopes were investigated using particle-particle random phased approximation PPRPA code version 1, 2015 (Taqi, 2016) in the presence of MSDI and N-N interactions. Our approach assumes that the low-lying states of ^{102}Sn, ^{110}Sn, ^{116}Sn, ^{120}Sn, and ^{122}Sn by acting two-particle operators on a correlated core ^{100}Sn, ^{108}Sn, ^{114}Sn, ^{118}Sn, and ^{120}Sn, respectively. A comparison had been made between our theoretical predictions and the recent available experimental data.

II. Theory

The collective excited states of multipolarity J and isospin T for the $A + 2$ systems can be constructed by operating $Q^{\dagger}_{\omega,JT}$ on the core $|0\rangle$ (Ring and Schuck, 1980),

$$Q^{\dagger}_{\omega}\left|0\right. = \left|A+2,\omega,JT\right. = \left(\sum_{m\le n} X^{\omega,JT}_{mn} a^{\dagger}_m a^{\dagger}_n - \sum_{i\le j} Y^{\omega,JT}_{ij} a^{\dagger}_i a^{\dagger}_j\right)\left|A,0\right. \quad (1)$$

where mn and ij represents orbits above and below the Fermi level, respectively. In pp RPA, two types of variations $\delta Q_{\omega,JT}\left|0\right.$ are available, they are: $a^{\dagger}_m a^{\dagger}_n\left|0\right.$ and $a^{\dagger}_i a^{\dagger}_j\left|0\right.$ which gives two sets of the equation of motion (Rowe, 1970; Ring and Schuck, 1980),

$$\left.\begin{aligned} RPA\left|\left[a_n a_m\left[H,Q^{\dagger}_{\omega,JT}\right]\right]\right|RPA &= E_x RPA\left|\left[a_n a_m, Q^{\dagger}_{\omega,JT}\right]\right|RPA \\ RPA\left|\left[a_i a_j\left[H,Q^{\dagger}_{\omega,JT}\right]\right]\right|RPA &= E_x RPA\left|\left[a_i a_j, Q^{\dagger}_{\omega,JT}\right]\right|RPA \end{aligned}\right\} \quad (2)$$

within the quasi-boson approximation, the amplitudes $X^{\omega,JT}_{mn}$. and $Y^{\omega,JT}_{ij}$ for $\left|A+2\ \omega\ JT\right.$. systems approximate as follows,

$$\left.\begin{aligned} X^{\omega,JT}_{mn} &= A,0|a_n a_m|A+2,\omega,JT \cong HF\left|\left[a_n a_m, Q^{\dagger}_{\omega,JT}\right]\right|HF \\ Y^{\omega,JT}_{ij} &= A,0|a_i a_j|A+2,\omega,JT \cong HF\left|\left[a_i a_j, Q^{\dagger}_{\omega,JT}\right]\right|HF \end{aligned}\right\} \quad (3)$$

The compact matrix of pp RPA can be written as (Ring and Schuck, 1980),:

$$\begin{pmatrix} A^{\omega,JT}_{mnm'n'} & B^{\omega,JT}_{mnij} \\ B^{\dagger,\omega,JT}_{mnij} & C^{\omega,JT}_{iji'j'} \end{pmatrix}\begin{pmatrix} X^{\omega,JT}_{mn} \\ Y^{\omega,JT}_{ij} \end{pmatrix} = E_x \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} X^{\omega,JT}_{mn} \\ Y^{\omega,JT}_{ij} \end{pmatrix} \quad (4)$$

$$\left.\begin{aligned} A^{\omega,JT}_{mnm'n'} &= \left(\varepsilon_m+\varepsilon_n\right)\delta_{mm'}\delta_{nn'} + V_{mnm'n'} \\ C^{\omega,JT}_{iji'j'} &= \left(\varepsilon_m+\varepsilon_n\right)\delta_{mm'}\delta_{nn'} + V_{mnm'n'} \\ B^{\omega,JT}_{mnij} &= -V_{ijmn} \end{aligned}\right\} \quad (5)$$

Where E_x and ε are the excited and single-particle energies, respectively, while $V_{mnm'n'}$ is the two-particle matrix element of the effective interaction. If sub matrices $B^{\omega,JT}_{mnij}$ and $C^{\omega,JT}_{iji'j'}$ set equal to zero, then the RPA equation reduces to the TDA equation.

The antisymmetric matrix elements of the MSDI used in this work have the form of the following (Brussaard and Glaudemans, 1977),:

$$\begin{aligned} V^{JT}_{ab,cd} = \frac{1}{2}A_T\left(-1\right)^{n_a+n_b+n_c+n_d} \times \\ \sqrt{\frac{(2j_a+1)(2j_b+1)(2j_c+1)(2j_d+1)}{(1+\delta_{ab})(1+\delta_{cd})}} \times \\ \left\{\begin{aligned} &\left(-1\right)^{j_a+j_b+j_c+j_d}\begin{pmatrix} j_a & j_b & J \\ \frac{1}{2} & \frac{-1}{2} & 0 \end{pmatrix}\times\begin{pmatrix} j_c & j_d & J \\ \frac{1}{2} & \frac{-1}{2} & 0 \end{pmatrix} \\ &\left[1-\left(-1\right)^{J+T+\mathrm{l}_c+\mathrm{l}_d}\right]-\begin{pmatrix} j_a & j_b & J \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} j_c & j_d & J \\ \frac{1}{2} & \frac{1}{2} & -1 \end{pmatrix} \\ &\left[1+\left(-1\right)^{T}\right]+\left\{\left[2T\left(T+1\right)-3\right]B+C\right\}\delta_{ac}\delta_{bd} \end{aligned}\right\} \end{aligned} \quad (6)$$

with

where B, C, A_0, and A_1 are the strength parameters of the (MSDI).

III. Results and Discussions

In the present study, the nuclear structures of Tin isotopes: ^{102}Sn,^{110}Sn, ^{116}Sn, ^{120}Sn, and ^{122}Sn are studied in the framework of pp TDA and RPA using PPRPA code version 1, 2015 (Taqi, 2016). The Hamiltonian is diagonalized within the model space including {$1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$} orbits, using MSDI and N-N interaction. The models assume that the low-lying states of ^{102}Sn are obtained by acting pp operator on a correlated ^{100}Sn core, of ^{110}Sn are obtained by acting pp operator on a correlated core of ^{108}Sn isotope, of ^{116}Sn are obtained by acting pp operator on a correlated core of ^{114}Sn isotope, of ^{120}Sn are obtained by acting pp operator on a correlated core of ^{118}Sn isotope, and of ^{122}Sn are obtained by acting

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pp operator on a correlated core ^{120}Sn isotope. The single particle energies are given in Table I.

The calculated *pp* TDA eigenvalues for ^{102}Sn that were calculated using N-N interactions and MSDI along with the corresponding experimental data of energy levels (https://www.nndc.bnl.gov) are tabulated in Table II and are also plotted in Fig. 1.

Few excited states with $J^{\pi} = 2^{+}$, 4^{+}, and 6^{+} are known from experiments. For instance, the first 2^{+} experimental excited states are obtained at 1.472 MeV, the values of *pp* TDA with MSDI were found at 1.848 MeV, and the same state was found at 1.934 MeV with the N-N interaction.

The calculated eigenvalues and experimentally known energy levels for ^{110}Sn, ^{116}Sn ^{120}Sn, and ^{122}Sn are tabulated in Tables III-VI and are also plotted in Figs. 2-5. The *pp*

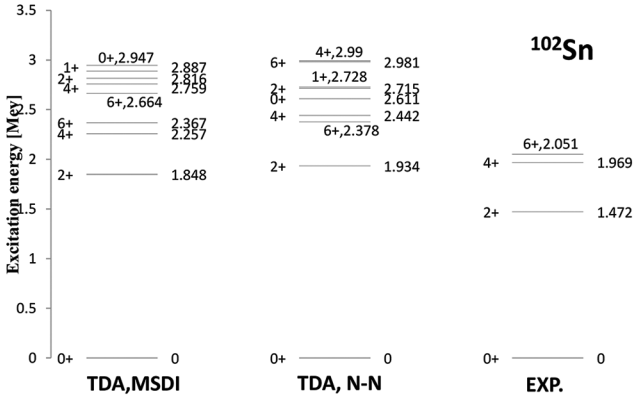

Fig. 1. Energy levels scheme of ^{102}Sn isotope using particle-particle Tamm-Dancoff approximation with modified surface delta interaction and neutron-neutron interactions in comparison with the experimental energies (https://www.nndc.bnl.gov).

TABLE I
SINGLE PARTICLE ENERGIES

ORBITS (*NLJ*)	S.P. ENERGY (MEV)
$1g_{7/2}$	−5.6014
$2d_{5/2}$	−5.2819
$2d_{3/2}$	−3.7090
$3s_{1/2}$	−3.7077
$1h_{11/2}$	−3.9843

TABLE II
THE CALCULATED LOW-LYING STATES (MEV) OF ^{102}SN USING MSDI AND N-N INTERACTIONS FOR THE *PP* TDA IN COMPARISON WITH THE EXPERIMENTAL DATA (HTTPS://WWW.NNDC.BNL.GOV)

J^{π}	EXP.	J^{π}	TDA, MSDI	J^{π}	TDA, N-N
0^{+}	0	0^{+}	0	0^{+}	0
2^{+}	1.472	2^{+}	1.848	2^{+}	1.934
4^{+}	1.969	4^{+}	2.257	6^{+}	2.378
6^{+}	2.051	6^{+}	2.367	4^{+}	2.442
		6^{+}	2.664	0^{+}	2.611
		4^{+}	2.759	2^{+}	2.715
		2^{+}	2.816	1^{+}	2.728
		1^{+}	2.887	6^{+}	2.981
		0^{+}	2.947	4^{+}	2.99

N-N: Neutron-neutron, *pp* TDA: Particle-particle Tamm-Dancoff Approximation, MSDI: Modified surface delta interaction

TABLE III
THE CALCULATED LOW-LYING STATES (MEV) OF ^{110}SN USING MSDI AND N-N INTERACTIONS FOR *PP* TDA AND *PP* RPA IN COMPARISON WITH THE EXPERIMENTAL DATA (HTTPS://WWW.NNDC.BNL.GOV)

J^{π}	EXP.	J^{π}	TDA, MSDI	J^{π}	TDA, N-N	J^{π}	RPA, N-N	J^{π}	RPA, MSDI
0^{+}	0	0^{+}	0	0^{+}	0	0^{+}	0	0^{+}	0
2^{+}	1.212	1^{+}	0.548	1^{+}	0.948	1^{+}	1.008	1^{+}	0.607
4^{+}	2.197	2^{+}	1.048	2^{+}	1.137	2^{+}	1.213	2^{+}	1.121
2^{+}	2.546	4^{+}	1.21	4^{+}	1.493	4^{+}	1.571	4^{+}	1.287
4^{+}	2.695	2^{+}	2.428	2^{+}	2.641	2^{+}	2.716	2^{+}	2.501
6^{+}	3.335	4^{+}	2.588	4^{+}	2.788	4^{+}	2.865	4^{+}	2.664
7^{+}	4.004	11^{+}	3.45	11^{+}	3.849	11^{+}	3.927	11^{+}	3.527
8^{+}	4.138	7^{+}	3.685	7^{+}	4.084	7^{+}	4.163	7^{+}	3.762
10^{+}	4.317	6^{+}	3.777	6^{+}	4.223	6^{+}	4.301	6^{+}	3.854
		8^{+}	3.815	8^{+}	4.295	8^{+}	4.374	8^{+}	3.893
		10^{+}	3.846	10^{+}	4.362	10^{+}	4.374	10^{+}	3.923
		0^{+}	4.29	0^{+}	4.762	0^{+}	4.838	0^{+}	4.366
		2^{+}	4.437	2^{+}	4.928	2^{+}	5.006	2^{+}	4.514

pp TDA: Particle-particle Tamm-Dancoff Approximation, *pp* RPA: Particle-particle Random Phase Approximation, MSDI: Modified surface delta interaction, N-N: Neutron-neutron

TABLE IV
THE CALCULATED LOW-LYING STATES (MEV) OF ^{116}SN USING MSDI AND N-N INTERACTIONS FOR *PP* TDA AND *PP* RPA IN COMPARISON WITH THE EXPERIMENTAL DATA (HTTPS://WWW.NNDC.BNL.GOV)

J^{π}	EXP	J^{π}	RPA, MSDI	J^{π}	RPA, N-N	J^{π}	TDA, MSDI	J^{π}	TDA, N-N
0^{+}	0	0^{+}	0	0^{+}	0	0^{+}	0	0^{+}	0
2^{+}	1.293	2^{+}	1.531	2^{+}	1.104	2^{+}	1.386	2^{+}	0.905
0^{+}	2.027	5^{+}	1.788	0^{+}	1.567	5^{+}	1.636	0^{+}	1.349
2^{+}	2.112	4^{+}	1.807	4^{+}	1.603	4^{+}	1.654	4^{+}	1.386
4^{+}	2.391	6^{+}	1.871	5^{+}	1.64	6^{+}	1.718	5^{+}	1.421
0^{+}	2.545	2^{+}	2.289	6^{+}	1.769	2^{+}	2.138	6^{+}	1.551
2^{+}	2.65	3^{+}	2.319	2^{+}	2.076	3^{+}	2.168	2^{+}	1.858
2^{+}	2.843	0^{+}	2.378	2^{+}	2.141	0^{+}	2.225	2^{+}	1.924
3^{+}	2.996	2^{+}	2.385	3^{+}	2.171	2^{+}	2.231	3^{+}	1.953
6^{+}	3.032	1^{+}	2.403	0^{+}	2.203	1^{+}	2.248	0^{+}	1.998
		2^{+}	2.533	1^{+}	2.254	2^{+}	2.378	1^{+}	2.034
		0^{+}	2.534	2^{+}	2.465	0^{+}	2.379	2^{+}	2.245

pp TDA: Particle-particle Tamm-Dancoff Approximation, *pp* RPA: Particle-particle Random Phase Approximation, MSDI: Modified surface delta interaction, N-N: Neutron-neutron

TABLE V
THE CALCULATED LOW-LYING STATES (MEV) OF ^{120}SN USING MSDI AND N-N INTERACTIONS FOR *PP* TDA AND *PP* RPA IN COMPARISON WITH THE EXPERIMENTAL DATA (HTTPS://WWW.NNDC.BNL.GOV)

J^{π}	EXP.	J^{π}	TDA, MSDI	J^{π}	TDA, N-N	J^{π}	RPA, MSDI	J^{π}	RPA, N-N
0^{+}	0	0^{+}	0	0^{+}	0	0^{+}	0	0^{+}	0
2^{+}	1.171	1^{+}	0.671	1^{+}	0.619	1^{+}	0.758	1^{+}	0.745
0^{+}	1.875	2^{+}	1.102	2^{+}	0.695	2^{+}	1.24	2^{+}	0.865
4^{+}	2.194	9^{+}	1.187	4^{+}	1.133	9^{+}	1.343	4^{+}	1.32
5-	2.284	4^{+}	1.246	9^{+}	1.135	4^{+}	1.396	9^{+}	1.33
1^{+}	2.297	6-	1.267	0^{+}	1.194	6-	1.423	0^{+}	1.383
5-	2.54	6^{+}	1.307	6-	1.215	6^{+}	1.46	6-	1.41
6^{+}	2.685	8^{+}	1.344	6^{+}	1.297	8^{+}	1.499	6^{+}	1.488
6-	2.749	10^{+}	1.373	8^{+}	1.369	10^{+}	1.528	8^{+}	1.564
8^{+}	2.802	5-	1.458	5-	1.411	5-	1.614	5-	1.605
1^{+}	2.835	5-	1.496	10^{+}	1.436	5-	1.652	10^{+}	1.63
6-	2.844	6-	1.687	5-	1.444	6-	1.843	5-	1.639
10^{+}	2.902	1^{+}	1.85	6-	1.667	1^{+}	2.001	6-	1.862
		0^{+}	1.904	1^{+}	1.798	0^{+}	2.058	1^{+}	1.988

pp TDA: Particle-particle Tamm-Dancoff Approximation, *pp* RPA: Particle-particle Random Phase Approximation, MSDI: Modified surface delta interaction, N-N: Neutron-neutron

RPA results are plotted in columns (second and third) for both interactions MSDI and N-N, respectively, while the *pp* TDA results are plotted in columns (fourth and fifth) for both interactions MSDI and N-N, respectively. The calculated results are compared with the experimental data (first column). The sequence of levels is very predictable based on both interactions.

TABLE VI
THE CALCULATED LOW-LYING STATES (MEV) OF ^{122}SN USING MSDI AND N-N INTERACTIONS FOR *PP* TDA AND *PP* RPA IN COMPARISON WITH THE EXPERIMENTAL DATA (HTTPS://WWW.NNDC.BNL.GOV)

J^π	EXP.	J^π	TDA, MSDI	J^π	TDA, N-N	J^π	RPA, N-N	J^π	RPA, MSDI
0^+	0	0^+	0	0^+	0	0^+	0	0^+	0
2^+	1.14	1^+	0.587	2^+	0.566	2^+	0.72	1^+	0.664
4^+	2.142	11^+	0.803	1^+	0.617	1^+	0.737	11^+	0.944
6^+	2.555	2^+	0.906	11^+	0.834	11^+	1.017	2^+	1.023
8^+	2.69	3^+	0.945	3^+	0.975	3^+	1.144	3^+	1.071
10^+	2.765	9^+	0.987	4^+	1.005	4^+	1.179	9^+	1.128
		5^+	1.009	9^+	1.017	9^+	1.2	5^+	1.145
		7^+	1.018	5^+	1.039	5^+	1.218	7^+	1.158
		4^+	1.043	7^+	1.048	7^+	1.231	4^+	1.176
		6^+	1.101	6^+	1.169	6^+	1.348	6^+	1.24
		8^+	1.136	8^+	1.24	8^+	1.423	8^+	1.277
		10^+	1.163	10^+	1.307	10^+	1.49	10^+	1.304

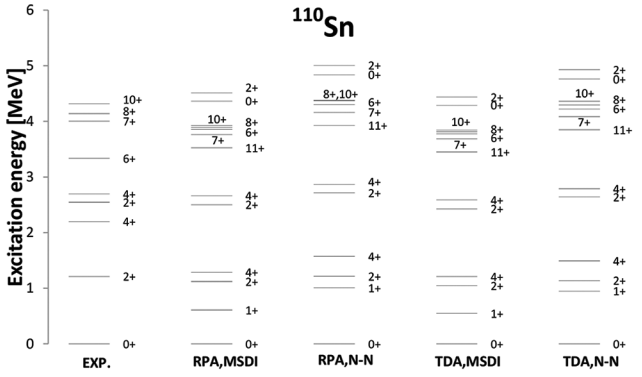

Fig. 2. Energy levels scheme of ^{110}Sn isotope using particle-particle Tamm-Dancoff approximation and particle-particle random phase approximation with modified surface delta interaction and neutron-neutron interactions in comparison with the experimental energies (https://www.nndc.bnl.gov).

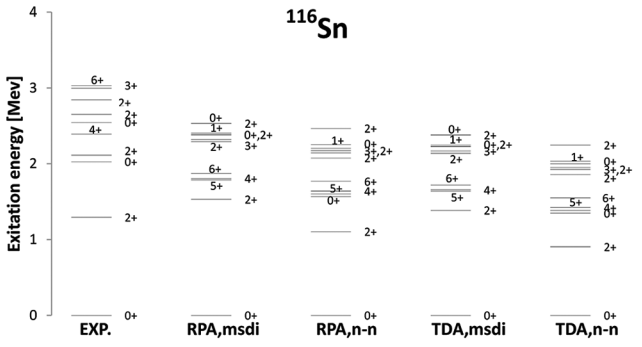

Fig. 3. Energy levels scheme of ^{116}Sn isotope using particle-particle Tamm-Dancoff approximation and particle-particle random phase approximation with modified surface delta interaction and neutron-neutron interactions in comparison with the experimental energies (https://www.nndc.bnl.gov).

The calculated low-lying states of ^{110}Sn and ^{116}Sn using MSDI and N-N interactions for *pp* TDA and *pp* RPA are in agreement with the experimental data, notably, 1^+ and 5^+ levels cannot be predicted experimentally or are not available.

The first experimental 2^+ excited states for ^{120}Sn occur at 1.171 MeV. The same state was found at 1.240 MeV when using *pp* RPA with the MSDI, but it was found at 0.865 MeV when using the N-N interaction. Similarly, the same state was found at 1.102 MeV when using *pp* TDA with the MSDI, but it was found at 0.695 MeV when using the N-N interaction.

There are only a few known experimentally excited states for ^{122}Sn (J^π= 2^+, 4^+, 6^+, 8^+, and 10^+), more states were predicted by *pp* TDA and *pp* RPA calculations for MSDI and N-N than by experimental evidence.

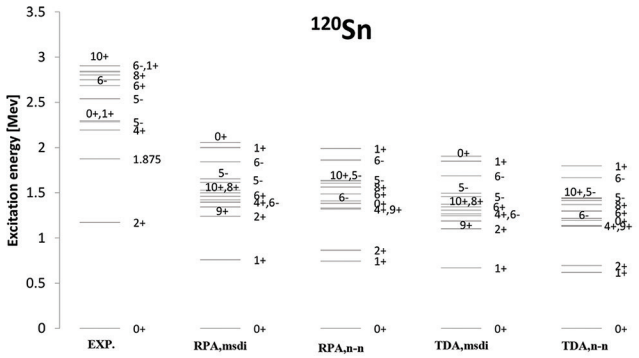

Fig. 4. Energy levels scheme of ^{120}Sn isotope using particle-particle Tamm-Dancoff approximation and particle-particle random phase approximation with modified surface delta interaction and neutron-neutron interactions in comparison with the experimental energies (https://www.nndc.bnl.gov).

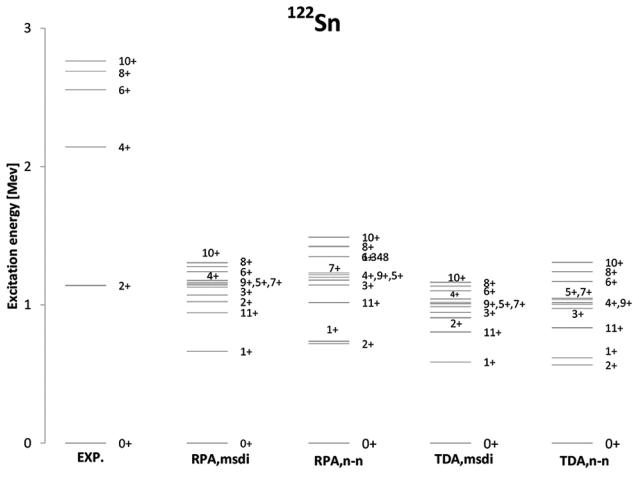

Fig. 5. Energy levels scheme of ^{122}Sn isotope using particle-particle Tamm-Dancoff approximation and particle-particle random phase approximation with modified surface delta interaction and neutron-neutron interactions in comparison with the experimental energies (https://www.nndc.bnl.gov).

IV. Conclusion

When the Hamiltonian is diagonalized in the presence of N-N interactions and MSDI, the calculated results of *pp* RPA and *pp* TDA are obtained in reasonable agreement to those of experimental data for the investigated Sn isotopes. Both the interactions predict very well the ordering of levels. However, for ^{102}Sn, the N-N interaction has the same experimental 2^+ state. The ground states of ^{110}Sn and ^{116}Sn are very nicely predicted. The calculated first 2^+excited state of ^{120}Sn using *pp* RPA with the MSDI obtained at 1.24 MeV is in good agreement with the experimental value at 1.171 MeV.

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